Insights from KiK-net Data: What Input Parameters Should Be Addressed to Improve Site Response Predictions?

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ABSTRACT

Site response models have been found to underpredict high-frequency ground motions in the aggregate, and we hypothesize these persistent biases are due to breakdowns in the one-dimensional (1D) site response assumptions and/or poorly characterized soil properties. Using data from Japan's Kiban-Kyoshin (KiK-net) network of vertical seismometer arrays, we test four physical explanations for this persistent bias by adjusting soil profiles and material parameters to address possible shortcomings in these model inputs. The physical adjustments include: (1) applying a depth-dependent shear-wave velocity (V_S) gradient within layers, (2) decreasing the small-strain damping ratio, (3) increasing the small-strain shear modulus, and (4) randomizing the V_S profile. Of these adjustments, we find that the usage of a depth-dependent V_S gradient most greatly reduces the high-frequency bias, and produces more realistic strain profiles and surficial ground motions at the test site in this study (FKSH11). With regards to 1D site response model improvement, this study suggests that greater attention should be paid to soil profiles and material parameters, as some of these physical adjustments are more successful at reducing model bias than changing the constitutive model type.

INTRODUCTION

Recent studies have used vertical seismometer arrays, such as those from Japan's Kiban-Kyoshin network (KiK-net), to advance our understanding of the bias and precision of site response models. The vast majority of site response models assume one-dimensional (1D) wave propagation and total-stress soil behavior. However, a number of studies have illustrated the limitations of 1D site response analyses across a range of ground motion levels (e.g., Kaklamanos et al. 2013, 2015; Kim and Hashash 2013; Zalachoris and Rathje 2015). This study follows upon recent efforts by the authors (Kaklamanos and Bradley 2015, 2016) in which nonlinear (NL) site response model predictions for 5626 ground motions at 114 KiK-net sites were calculated using DEEPSOIL (Hashash et al. 2016) and compared to observed ground motions and predictions from linear (L) and equivalent-linear (EQL) analyses in SHAKE (Ordoñez 2017). Using this large database of 1D site response model predictions, a variety of statistical analyses were performed to quantify the models' uncertainties.

In these prior studies, we found that the differences in accuracy are largest between the linear model and the other models; there are generally small differences between the equivalent-linear and nonlinear models. Another less intuitive result is the fact that all models display persistent underpredictions at short periods. Consider Figure 1, which depicts the period-dependent L, EQL, and NL model biases for 5%-damped pseudo-acceleration response spectra (PSA) for different levels of shear strain. The model bias is computed as $E\{\ln[PSA_{obs}(T)] - \ln[PSA_{pred}(T)]\}$, where $PSA_{obs}(T)$ and $PSA_{pred}(T)$ refer to the observed and predicted response spectra, respectively, at a given vibration period (T), and the mean is calculated across the entire ground-motion set. Two important trends are observed in this figure: (1) all models (L, EQL, and NL) offer persistent underpredictions (positive bias) at short spectral periods, and (2) the most severe bias occurs for small-strain motions. The use of a more complicated constitutive model (e.g., NL) does not resolve the issue of bias at short periods (high frequencies).

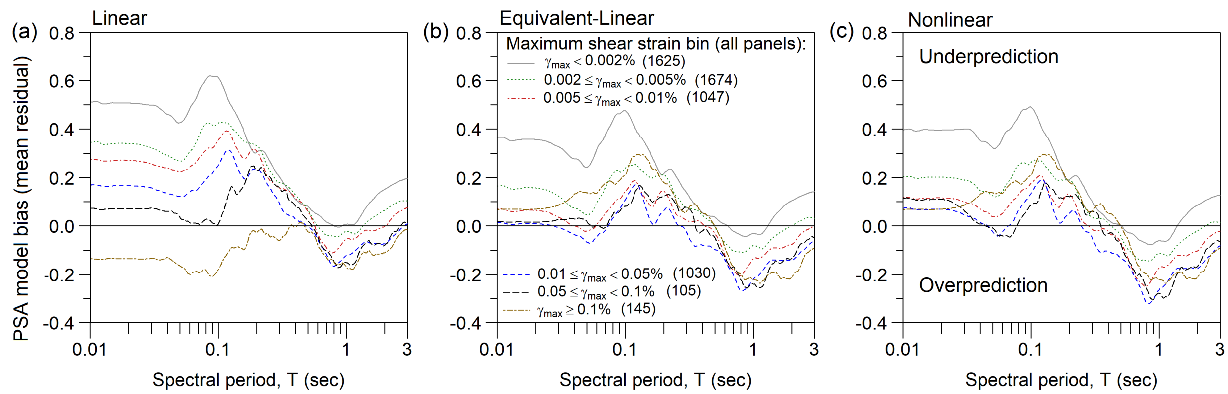

Figure 1. Model bias for PSA as a function of spectral period for different bins of maximum shear strain: (a) L, (b) EQL, and (c) NL. The numbers of ground motions in each bin are indicated in parentheses; the aggregate dataset includes 5626 ground motions at 114 sites.

Because this high-frequency bias is prevalent across all model types, we hypothesize that breakdowns in the 1D site response assumptions and/or poorly characterized soil properties are responsible for the persistent underpredictions. To test this hypothesis, we adjust the original soil profiles and material parameters to address possible shortcomings in the input parameter data and/or assumptions. Using an example site in the KiK-net database (station FKSH11), we perform the following adjustments: (1) apply a depth-dependent shear-wave velocity (V_S) gradient within layers, (2) decrease the small-strain damping ratio, (3) increase the small-strain shear modulus, and (4) randomize the V_S profile. Using station FKSH11 as an example, we describe the motivation for, and the mechanics of, the four physical adjustments; present results of the site response analyses in a physical and statistical context; and provide recommendations for determining soil profiles and constitutive model parameters in site response models. Future work will extend the analyses of these physical adjustments to ten KiK-net sites.

DATA AND METHODS

Vertical seismometer arrays, which have both surface and downhole seismometers, are a robust data source for evaluating site response models. The observed downhole record is used as the input motion to the site response model, and the motion is propagated through the soil profile (assuming 1D wave propagation; plane-strain deformation) to predict the ground motion at the surface. The Kiban-Kyoshin network (KiK-net) of vertical seismometer arrays in Japan (Okada et al. 2004), with its extensive database of ground-motion recordings, has been used in this study.

The subsurface data available for KiK-net sites include P- and S-wave seismic velocity profiles (measured from surface-downhole logging) and geologic profiles, which provide information on soil/rock type and geologic age. Recent assessments of California downhole array data by Afshari and Stewart (2015) and Li et al. (2018) generally find lesser bias at the California sites than the KiK-net sites, although the library of ground motions at the California sites is smaller.

The test site in this paper is KiK-net station FKSH11 (Latitude: 37.1976°N, Longitude: 140.3420°E), located in Fukushima prefecture on eastern Honshu island. The site consists of 35 m of sand and gravel over tuffaceous rock, and there exists a velocity inversion due to a layer of welded tuff between depths of 35 and 57 m. Figure 2 provides the V_S profile obtained from the KiK-net website, as well as the translated geologic log. The time-averaged shear-wave velocity in the upper 30 m (V_{S30}) is 240 m/s, and the profile extends to 115 m depth, where the downhole seismometer is installed. From the broader dataset of 114 sites, FKSH11 was selected because it satisfies the criteria of Thompson et al. (2012) with regards to the applicability of 1D site response models; has a somewhat interesting V_S profile (due to the sharp impedance contrast at 34 m and the velocity inversion); and has significant bias at high frequencies. Across all ground motions assessed at the site, the NL model underprediction bias for peak ground acceleration (PGA) is 0.462 natural logarithmic units (Kaklamanos and Bradley 2015); equivalently, the average ratio of predicted to observed PGA is 0.63 [exp(−0.462) = 0.630].

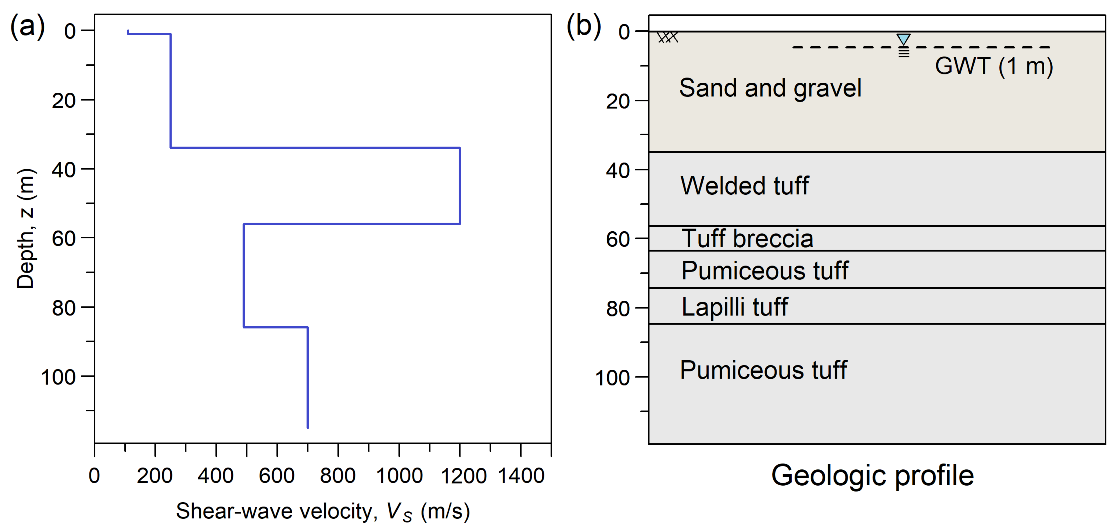

Figure 2. Profiles of (a) shear-wave velocity and (b) stratigraphy at KiK-net station FKSH11.

Site response analyses were performed using L, EQL, and NL models for 46 ground motions at FKSH11, representing records from 2001 through 2014 that meet the signal strength criteria of Thompson et al. (2012). For brevity, only the NL site response analysis results are discussed in this paper. The Zhang et al. (2005) modulus-reduction and damping curves were used as the target relations for the NL analyses, and the fitting was performed using the MRDF pressure-dependent hyperbolic model procedure (Phillips and Hashash 2009) in DEEPSOIL. The dynamic behavior of the soil layers was represented using the MKZ model (Matasovic and Vucetic 1993), and linear stress-strain behavior was assumed in all rock layers (corresponding to depths greater than 35 m). Site response analyses were performed for each of the four physical adjustments described in the following section, in addition to the original profile.

ADJUSTMENTS TO SOIL PROFILES AND CONSTITUTIVE MODEL PARAMETERS

Motivated in the context of the observed model biases, four physical hypotheses regarding V_S profiles and constitutive model parameters are tested. Kaklamanos and Bradley (2016) and Kaklamanos et al. (2017) showed that each site response model (L, EQL, and NL) exhibited bias at nearly all strain levels, with the strongest bias occurring at small strains. Because soil profiles and material parameter assumptions influence ground-motion predictions at all strain levels, attention to these parameters might help explain the persistent model biases.

Depth Dependence of Shear-Wave Velocity. First, we hypothesize that the V_S profiles provided on the KiK-net website may be too coarse, and that as a result, the impedance contrasts in the V_S profiles between adjacent layers may be excessively larger than those that exist in reality. For example, the KiK-net V_S profile at FKSH11 (Figure 2) is 110 m/s for depths 0–1 m and 250 m/s for depths 1–34 m. Due to increasing confining pressures, constant or increasing densities with depth should lead to an increase in V_S with depth in a given layer. In reality, it is highly unlikely that V_S is constant at 250 m/s over a 33-meter-thick layer so close to the surface. The oversimplified soil profile is a highly plausible reason for the underprediction of high frequencies, because it is the high frequencies that are caused by complex layering. At low frequencies, the thicknesses of the small layers (i.e., the layers that are perhaps missed in the simplified KiK-net profiles) are not large enough to affect the response, and therefore this source of bias is more pronounced at high frequencies. Another issue with coarse V_S profiles is that the impedance contrasts may be exaggerated. Impedance contrasts can have a profound impact on site response (Baise et al. 2016), and overly coarse profiles can produce unrealistically large strain localizations directly above the impedance contrast (Kaklamanos and Bradley 2016). These strain localizations can result in excessive dissipation of high-frequency energy.

The depth-dependent V_S hypothesis is tested as follows: within each layer, the constant value of V_S is replaced with a depth-dependent exponential gradient centered on the median V_S for the layer. Mathematically, the depth dependence of shear wave velocity within a layer is calculated as

$$V_s(z) = \overline{V_s}\left[\frac{\sigma_v'(z)}{\overline{\sigma_v'}}\right]^n , \quad (1)$$

where $V_s(z)$ = shear-wave velocity at a depth z in the profile, $\overline{V_s}$ = average shear-wave velocity throughout layer (constant), $\sigma_v'(z)$ = vertical effective stress at depth z in the layer, $\overline{\sigma_v'}$ = vertical effective stress at the layer midpoint, and n = stress exponent (1/4 for clays, silts, and sands; and 1/3 for gravels and rocks). These stress exponents are adapted from values in Lin et al. (2014). The result of Equation 1 is an exponential variation in V_S with depth throughout the layer.

Small-Strain Damping Ratio. Kaklamanos et al. (2017) observed that all models offer their most severe underpredictions for small-strain motions, and therefore we hypothesize that the assumed small-strain damping in the constitutive models may be too large. Note, however, that this is an inference based solely on the statistical observation of underprediction at high frequencies. Other studies (e.g., Afshari and Stewart 2015, Zalachoris and Rathje 2015, Cabas et al. 2017) have found that laboratory-based models for small-strain damping are smaller than field estimates of damping, because laboratory-based models cannot capture more complex energy dissipation phenomena such as scattering of seismic waves.

In this adjustment, the revised small-strain damping ratio, D'_{min}, is calculated from the original small-strain ratio D_{min} by $D'_{min} = R_{min} \cdot D_{min}$, where $R_{min} \in [0,1]$ is a reduction factor to be applied to the original small-strain damping ratio. In this study, we assume $R_{min} = 0.5$ (selected on the basis of reducing the observed model biases, rather than a physical or theoretical basis), so that the original small-strain damping ratio is reduced by half. In addition to adjusting the small-strain damping ratio, the target damping curve must also be adjusted. At small strains ($\gamma \leq 0.001\%$), we multiply the damping curve by the small-strain reduction factor R_{min}. At larger strains ($\gamma \geq 0.01\%$), we do not assume any reduction in the damping curve (that is, the reduction factor is unity). At intermediate strains ($0.001\% < \gamma < 0.01\%$), the reduction factor increases in a log-linear manner from R_{min} at $\gamma = 0.001\%$ to unity at $\gamma = 0.01\%$.

Small-Strain Shear Modulus. Third, we focus on another source of potential bias in the shear-wave velocity profile: underestimation of the small-strain shear modulus G_{max}. The lowest shear strain bin used by Kaklamanos and Bradley (2017), $\gamma_{max} < 0.002\%$, is the bin with the strongest bias. We hypothesize that field measurements may slightly underestimate the true G_{max} because larger strains may actually be incurred in the soil during testing. The small-strain shear modulus G_{max} and shear-wave velocity V_S are related by the physical equation $G_{max} = \rho V_S^2$, where ρ is the soil density. Kramer (1996) and others suggest that low-strain in-situ dynamic testing may induce shear strains of up to ~0.001% in the soil. Consider the modulus reduction curve in Figure 3 for the surficial layer at FKSH11. If shear strains of ~0.001% are used in the measurement of V_S (and hence the calculation of G_{max}), then this level of strain might actually induce a small level of nonlinearity in the soil. Because the shear modulus G measured in the field might be slightly less than the true G_{max} (that is, $G/G_{max} < 1$), we have increased G_{max} by 10% in all analyses. This correction implies that the V_S profiles be scaled by a factor $\sqrt{1.1} = 1.0488$, or increased by approximately 5%.

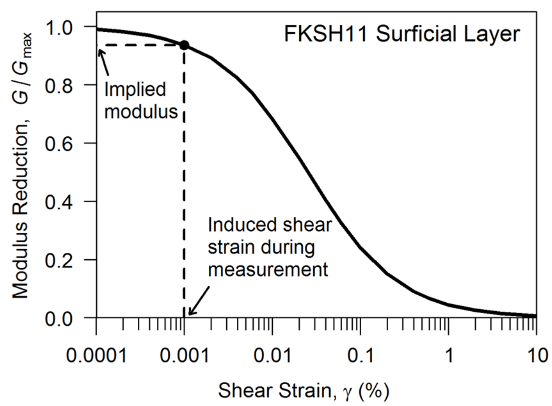

Figure 3. Illustration of the potential measurement bias in G_{max}, using the assumed modulus reduction curve of the surficial layer at FKSH11. If the surface-downhole test induces 0.001% strain in the soil, then the associated shear modulus is actually less than the true G_{max}.

Randomization of Shear-Wave Velocity. One-dimensional site response models cannot account for lateral variations in subsurface geology that may strongly influence ground motions at many sites. Three-dimensional (3D) effects (such as lateral variations in subsurface seismic velocities) can scatter and/or focus seismic waves, producing variations that cannot be captured by 1D wave propagation. The influence of lateral variations in subsurface geology is especially pronounced at high frequencies, where nonlinear soil behavior exerts its greatest influence. We hypothesize that 1D site response models may not accurately represent 3D subsurface heterogeneity, and that adding uncertainty to the V_S profiles may help better capture variability in soil properties. The most straightforward manner to incorporate 3D variability into a 1D site response model is by using a baseline velocity model and adding randomness to that layering. To test this hypothesis at each site, five randomized profiles (realizations) are generated using the commonly used Toro (1995) model for V_S uncertainty. Site response analyses are performed for each randomized profile, and then the median ground motion predictions of the randomized profiles are analyzed. Although Teague and Cox (2016) found that randomized profiles from the Toro (1995) method produce poor matches to the site dispersion curve, no dispersion curves are available for KiK-net sites.

Figure 4 provides the adjusted V_S profiles and corresponding theoretical 1D linear amplification spectra for the alternative physical hypotheses at FKSH11.

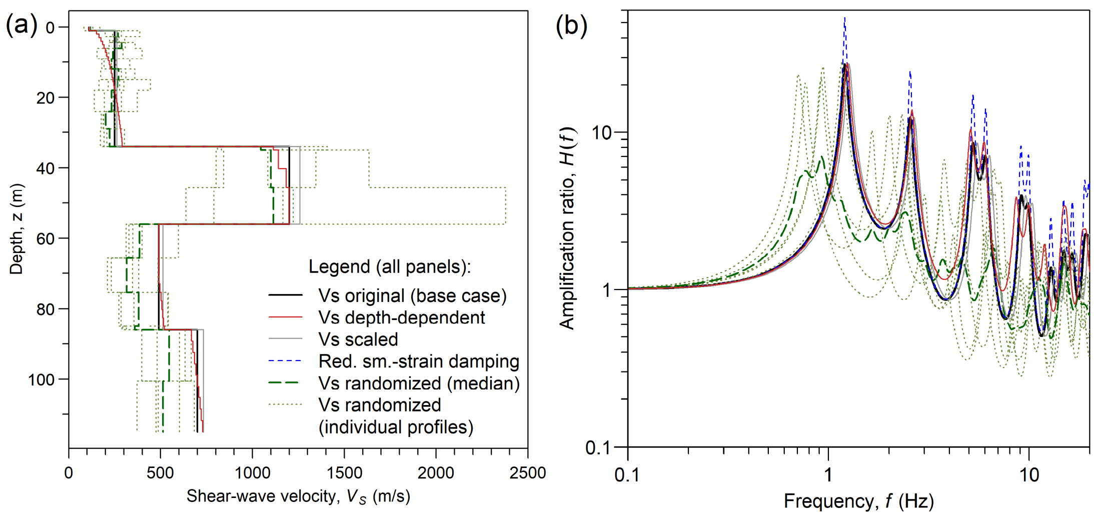

Figure 4. Results of the adjustments to the soil profiles and constitutive model parameters at FKSH11; (a) V_S profiles, and (b) theoretical 1D linear amplification spectra for the alternative physical hypotheses.

RESULTS: ANALYSIS OF A MODERATE GROUND MOTION

In an attempt to establish a physical context for the results, Figure 5 presents the ground motion predictions for a representative moderate record at FKSH11: the M_w 6.4 earthquake of 31 July 2011 (PGA = 0.117*g*). A moderate record has been selected for this discussion so that the results remain as general as possible. First, there are significant deviations in the profiles of maximum shear strain in Figure 5h. Many sites have strong localizations of shear strain directly above the interface of an impedance contrast; at FKSH11, this occurs at 34 m depth. As observed in the figure, the incorporation of depth-dependent V_S profiles results in smoother strain profiles and less dramatic strain localizations, potentially reducing the artificial dissipation of high frequencies at these points in the profile (and resulting in less overall underprediction bias, as observed in Figure 5f). As seen in the strain profiles in Figure 5h, there is an additional distinction for the depth-dependent V_S profiles: there is generally decreasing shear strain with depth within a layer (which seems more consistent with physical expectations), whereas the other hypotheses often result in increasing shear strain with depth within a layer (which is not as realistic).

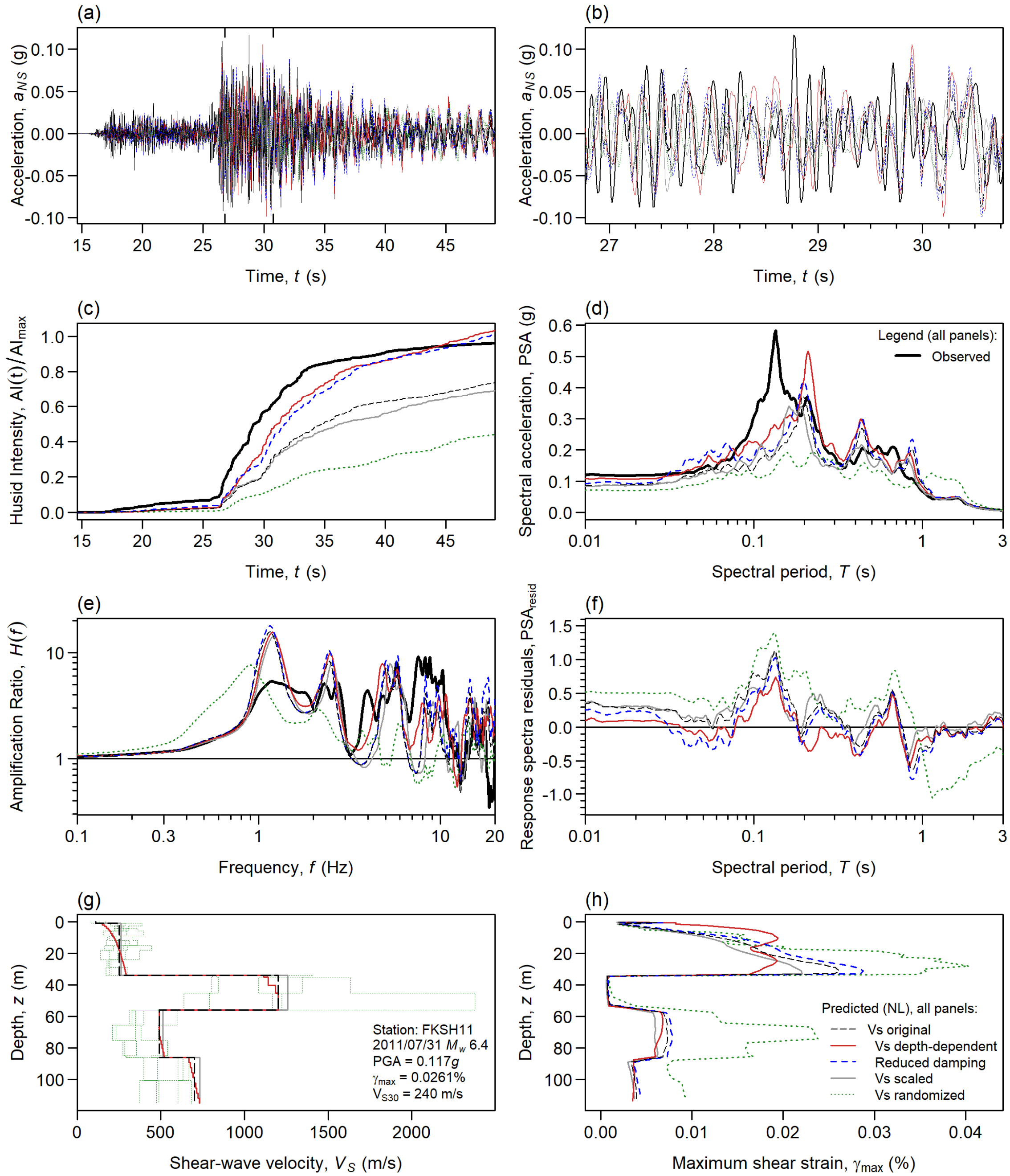

Figure 5. Example site response observations and predictions for a moderate ground motion at FKSH11, the M_w 6.4 earthquake of 31 July 2011: (a) acceleration time series throughout whole record, (b) acceleration time series centered at maximum acceleration (corresponding to the tick marks in panel (a)), (c) Husid plot (accumulation of Arias Intensity throughout the time series), (d) pseudo-acceleration response spectra, (e) surface/downhole amplification ratios, (f) pseudo-acceleration response spectra residuals (in natural logarithmic space), (g) shear-wave velocity profile, and (h) maximum shear strain profile. Different curves in the figure correspond to the four different physical adjustments (depth-dependent V_S, randomized V_S, reduced small-strain damping, and increased small-strain G$_{max}$ [scaled V_S]).

Like the depth-dependent V_S gradient, the reduction of the small-strain damping ratio tends to improve the results at high frequencies (reduced underprediction bias). The Husid plots in Figure 5c for the depth-dependent V_S and reduced small-strain damping both are considerably close to the observed Husid plot. This ground motion record generally indicates a minimal difference for the scaled V_S profile, which tends to mimic the results for the original V_S profile (although with slightly smaller ground motions). The randomized V_S profile offers the smallest prediction of surficial ground motions, and this is most predominantly observed in the Husid plot in Figure 5c and amplification spectra in Figure 5e. The randomization results in greater energy dissipation at depth, not unlike the scattering of seismic waves that 3D effects often produce. The predicted ground motions from the randomized profile are therefore physically consistent with the behavior we are trying to replicate. However, the randomized profile offers a greater underprediction bias (greater positive residuals) than the other physical hypotheses.

RESULTS: MODEL BIAS

Figure 6 displays the NL model bias (mean residual) as a function of spectral period for the alternative physical hypotheses at FKSH11 based on the 46 evaluated recordings at this site. The depth-dependent V_S gradient and revision of small-strain damping show the greatest reductions in bias. The usage of the depth-dependent V_S gradient reduces the artificial strain localizations at impedance contrasts and provides a more realistic profile of γ_{max} versus depth. The increased small-strain shear modulus (V_S scaled) leads to small changes in the shear-wave velocity profile, and therefore produces insignificant differences in the site response predictions. For FKSH11, the randomized profiles do not present a significant improvement from the original profile; the randomized profiles result in reduced amplitudes, and the low-Vs realizations tend to skew the median results downward.

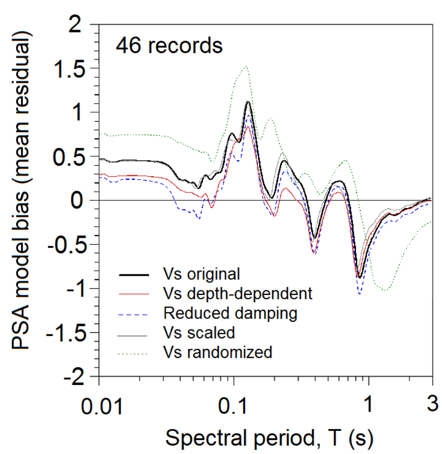

Figure 6. NL model bias (mean residual) as a function of spectral period for the alternative physical hypotheses, using the 46 ground motions at FKSH11.

CONCLUSIONS

Four physical hypotheses have been tested at KiK-net site FKSH11 to explain the underlying bias observed in 1D site response models at high frequencies. We find that the reduction of the small-strain damping ratio and the usage of a depth-dependent V_S gradient most greatly reduce the high-frequency bias; that the randomized V_S profiles lead to greater underpredictions at high frequencies; and that the adjustment of the small-strain shear modulus has a minimal effect. Of these adjustments, we find that the usage of a depth-dependent V_S gradient produces the most realistic strain profiles and surficial ground motions. Although the reduction of the small-strain damping ratio also reduces the bias, there is less of a physical basis for this adjustment; therefore, it is possible that this adjustment is resolving some other source of high-frequency bias (i.e. some sort of confounding error), such as breakdowns in the 1D assumption or overly coarse V_S profiles. In future work, we will extend these physical adjustments to a broader range of KiK-net sites, and will assess the combined effect of simultaneously applying a depth-dependent V_S gradient and reduced small-strain damping ratio (with varying levels of R_{min}). With regards to 1D site response model improvement, this study suggests that greater attention should be paid to soil profiles and material parameters, as some of these physical adjustments are more successful at reducing model bias than changing the model type.

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